BULK SOLID FLEXURE RESISTANCE

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ABSTRACT

When a bulk solid is transported on a belt conveyor, flexure resistance occurs between successive idler sets as the bulk solid undergoes transverse and longitudinal displacement due to the sag of the belt. This paper discusses the factors that influence the bulk solid flexure and describes a method to calculate the bulk solid flexure resistance coefficient. The influence of the internal friction angle of the bulk solid, idler spacing, belt sag and belt speed are demonstrated through a number of examples which provide the conveyor designer with an appreciation of the influence of each parameter on the energy losses due to bulk solid flexure.

1. INTRODUCTION

As the length of belt conveying systems increase it becomes ever more important to accurately calculate the motion resistances at the design stage, with the view of minimising these resistances to improve the efficiency of the installation. The motion resistances that occur along the length of the conveyor are known as the main resistances and include the belt and bulk solid flexure resistance, the rotational resistance of the idler rolls and the indentation rolling resistance of the conveyor belt. While it is widely accepted that the indentation rolling resistance is the major contributor to the motion resistance of long horizontal belt conveyors, the next highest component in most systems is the resistance due to the flexure of the bulk solid. This paper specifically discusses the nature of bulk solid flexure and highlights the most influential parameters.

Limited research has been conducted in the area of bulk solid flexure resistance primarily due to the perception that this resistance contributes little to the total resistance. Additionally, the bulk solid properties are generally considered a constraint rather than a variable parameter by the conveyor designer. While the conveyor designer has little control over the properties of the bulk solid being conveyed, the influence of these properties on the main resistances in some instances are significant and should not be overlooked. The bulk solid flexure resistance is consistently the second largest of the main resistances, as noted by Hager and Hintz [1] (see Figure 1), and may exceed the indentation rolling resistance in the case of wide conveyor belts, as noted by Spaans [2]. The belt and bulk solid properties influence the bulk solid flexure resistance to varying degrees, as does the belt speed, belt sag and idler spacing, thereby offering potential reduction with informed design.

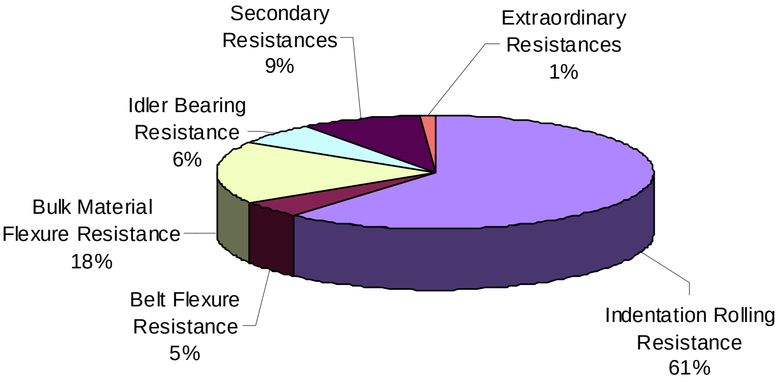

Figure 1: Breakdown of motion resistances for a belt conveyor approximately 1000m long (Ref: Hager and Hintz [1]).

This paper discusses the factors that influence the bulk solid flexure resistance and provides an overview of a numerical method developed by the author to calculate the loss coefficient. Results showing the influence of important conveyor parameters, such as the internal friction angle of the bulk solid, belt sag, belt speed and idler spacing are detailed so as to highlight the effect of each.

2. BULK SOLID FLEXURE

Bulk solid flexure resistance occurs between successive idler sets as the bulk solid undergoes transverse and longitudinal displacement due to belt sag. As the belt progresses from one idler set to the next the bulk solid undergoes cyclic expansion and contraction in the transverse direction, in addition to variation in height in the longitudinal direction, as shown in Figure 2. The relative movement results in energy losses due to the internal friction of the bulk solid.

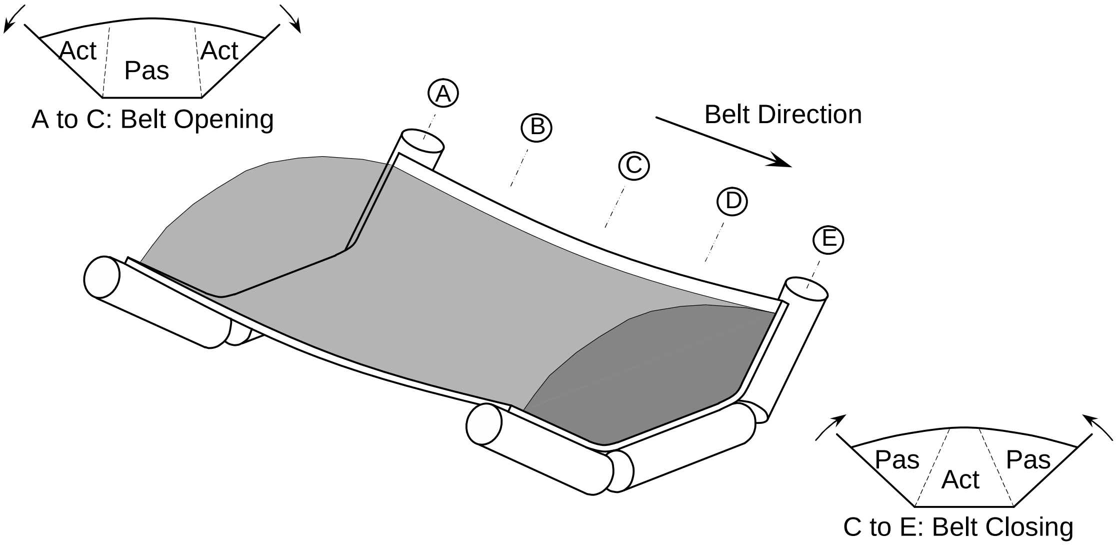

Figure 2: Induced active and passive stress states for a loaded conveyor belt.

For the purpose of analysis and explanation, bulk solid flexure can be considered to consist of both transverse and longitudinal resistance components. When the belt is supported by an idler set, as indicated by positions A and E, the bulk solid is forced to conform to the troughing profile, resulting in transverse compressive stresses. As the belt moves to position B, the troughed belt opens under the action of gravity allowing the bulk solid to relax transversely forming an active stress state. Longitudinally, however, the bulk solid is undergoing compressive stress due to the contraction of the bulk solid arising from the sag of the belt. Upon reaching approximately 50% to 60% of the idler spacing, as indicated by position C, the stress states theoretically reverse. A passive stress state is induced in the transverse direction due to the compressive stresses caused by the narrowing profile of the belt, while the bulk solid in the longitudinal direction dilates generating an active stress state as it moves away from the point of maximum sag. The cyclic transverse and longitudinal flexure of the bulk solid results in flexure losses due to internal friction and friction at the belt and bulk solid interface.

3. ANALYTICAL AND NUMERICAL MODELS

In order to calculate the flexure resistance of the bulk solid the forces generated from the relative movement of the bulk solid need to be resolved, taking into account the properties of the bulk solid and the conveyor belt. The magnitude of the belt deflection, troughing configuration and belt speed each contribute to the amount of belt and bulk solid flexure, and therefore, the flexure resistance. Bulk solid properties including bulk density, bulk solid surcharge angle, internal friction and friction at the belt interface determine the pressure distribution acting on the belt and the losses attributable to the relative movement of the bulk solid.

Spaans [2] was the first to provide an analytical model to calculate the flexure resistance of the bulk solid due to the cyclic transverse and longitudinal deformation. The transverse flexure resistance is modeled by calculating the difference between the work done during the opening and closing of the belt, as the belt moves between consecutive idler sets. The normal forces acting on the side idler rolls are calculated using a method developed by Krause and Hettler [3], who provide an analysis of the total force acting on the idler rolls due to the formation of active and passive stress states within the cross-section of bulk solid, as shown in Figure 3.

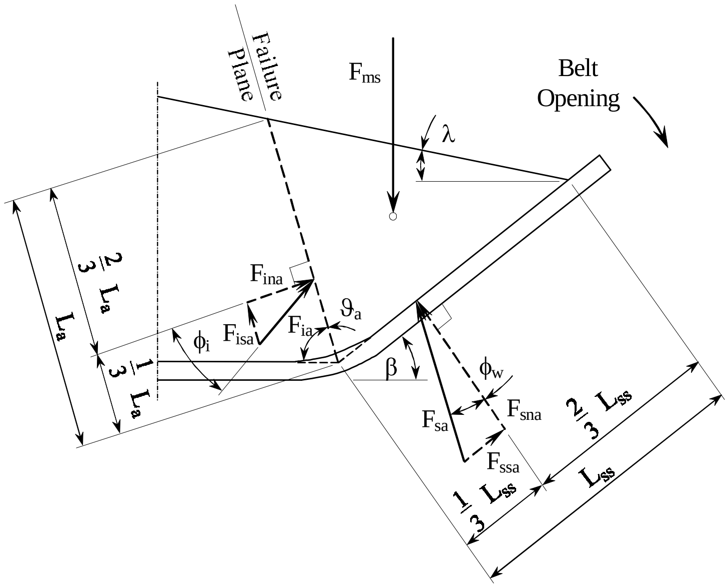

Figure 3: Force analysis for active stress case.

The transverse active pressure factor K_{ta}, for the opening conveyor belt was derived in terms of the troughing angle β, bulk solid internal friction angle ϕ_i, belt and bulk solid friction angle ϕ_w and the conveyor surcharge angle λ, and is given by

$$K_{ta} = \left[\frac{\sin(\beta + \phi_i) / \sin\beta}{\sqrt{\sin(\beta - \phi_w)} + \sqrt{\sin(\phi_i + \phi_w)\sin(\phi_i - \lambda) / \sin(\beta + \lambda)}} \right]^2 \qquad (1)$$

Similarly, the transverse passive pressure factor K_{tp}, for the closing conveyor belt is

$$K_{tp} = \left[\frac{\sin(\beta - \phi_i) / \sin\beta}{\sqrt{\sin(\beta + \phi_w)} - \sqrt{\sin(\phi_i + \phi_w)\sin(\phi_i + \lambda) / \sin(\beta + \lambda)}} \right]^2 \qquad (2)$$

The normal force per unit length F_{sn}, acting on the side idler roll due to the bulk solid is then approximated as

$$F_{sn} = \frac{1}{2}\rho g L_{ss}^{2} \frac{\left(K_{ta} + K_{tp}\right)}{2} cos\,\phi_{w} \tag{3}$$

Where ρ is bulk density, L_{ss} is the length of bulk solid in contact with the inclined side of the conveyor belt, and the active and passive stress states are assumed to act over half the idler spacing. Given the force acting the transverse flexure resistance is calculated from the difference between the work done during the opening and closing of the belt.

Spaans [2] also calculated the longitudinal flexure resistance. This analysis involved considering an elemental volume of bulk solid with vertical boarders on top of a flat belt. As the element moves between successive idler sets it undergoes compressive forces due to the belt sag, and since the bulk solid possesses internal friction, energy is absorbed in the bulk solid in the form of flexure resistance.

The numerical analysis discussed in this paper, is detailed by Wheeler [4], and adopts a similar approach to that of Spaans [2] by individually calculating the transverse and longitudinal components of the bulk solid flexure resistance. The analysis uses orthotropic plate mechanics to calculate the belt deflection to provide a means of predicting the flexure resistance due to the relative movement of the bulk solid. A similar approach to that of Krause and Hettler [3] is used to predict the active and passive stress states that are formed within the bulk solid as the belt opens and closes between successive idler sets. The pressure factors given by Eqns (1) and (2) are used, but rather than calculating the resultant normal force acting on the conveyor belt due to the induced stress states, the analysis calculates the pressure distribution over the surface area of the conveyor belt. The basis for the transverse model is pictured in Figure 4.

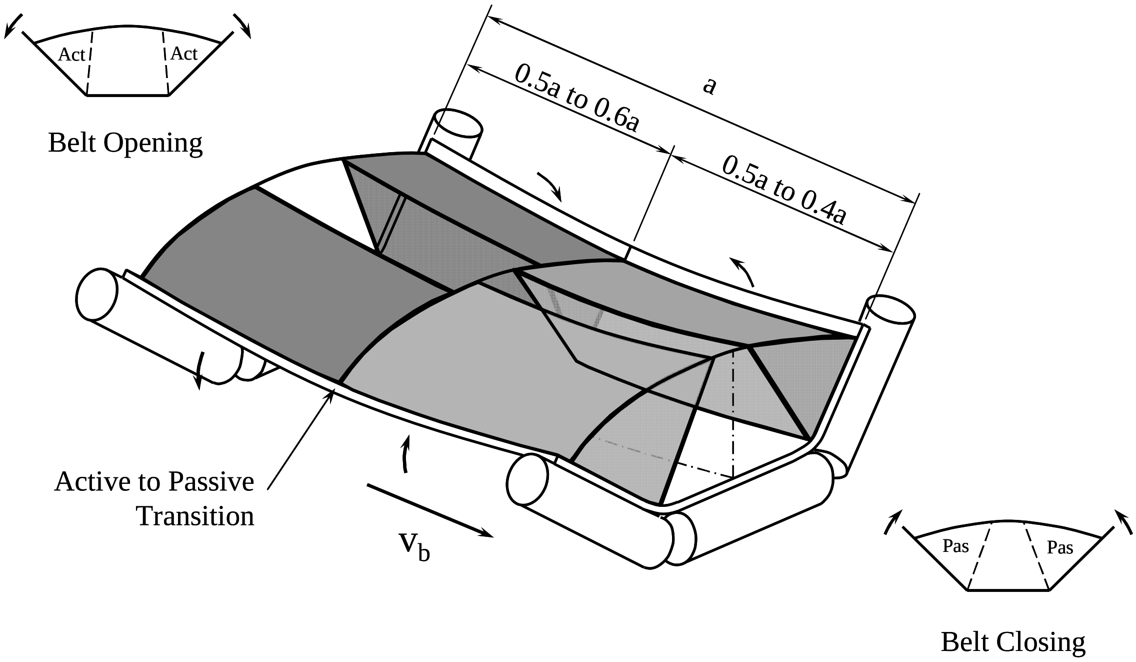

Figure 4: Transverse active and passive stress states that are formed within the bulk solid as the belt opens and closes between successive idler sets.

Figure 5 shows the model of the longitudinal flexural component which is calculated from the work done in deflecting the bulk solid acting along the centre of the belt directly above the centre idler roll. The segment analysed is determined by the shear planes set up in the bulk solid resulting from the induced stress states.

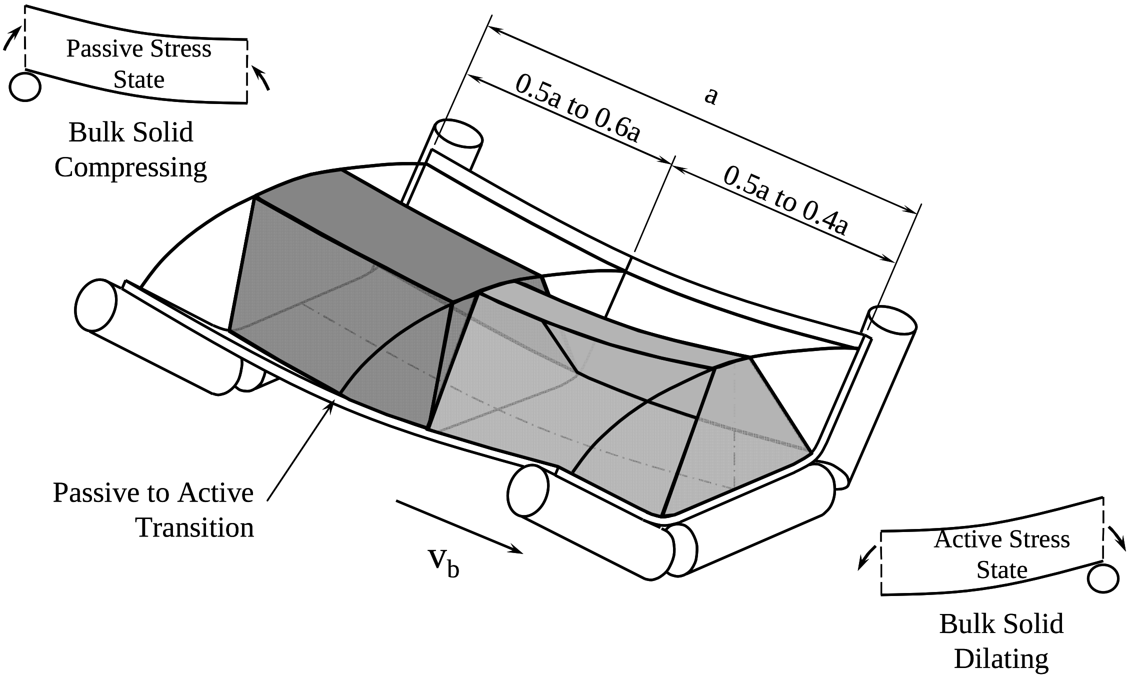

Figure 5: Longitudinal active and passive stress states that are formed within the bulk solid as the belt deflects longitudinally between successive idler sets.

An important aspect of the analysis is the allowance for the influence of belt speed. As indicated previously the transition between the stress states is assumed to occur at 50% to 60% of the idler spacing, a, as indicated by position C in Figure 2. The exact location of the transition is heavily dependant on the belt speed, v_b, since as the belt speed increases the transition, and therefore the point of maximum sag moves further away from the midpoint of the idler spacing. Typically at high belt speeds the transition will occur at 55 to 60% of the idler spacing. Since the bulk solid flexure resistance is calculated from the difference between the work done during each stress state, increasing belt speed has the effect of increasing bulk solid flexure resistance. To account for these dynamic effects an iterative procedure is employed. The procedure initially assumes that the transition occurs at the midpoint of the idler spacing and then with each iteration the profile of the belt alters as a result of the momentum of the moving bulk solid.

In addition to the numerical analysis described the author is currently utilising DEM(1) to simulate the movement of bulk solid as it is conveyed from one idler set to the next. The University of Newcastle is currently undertaking research using DEM code developed by CSIRO's(2) Mathematical and Information Services Division. CSIRO's Granular Flow Code is currently being used to simulate the moving bulk solid with the principal advantage being not having to specify the active and passive stress regions. The DEM code will be integrated with the conveyor belt deflection model and once again an iterative procedure will be employed to allow for momentum effects.

4. RESULTS AND DISCUSSION

To demonstrate the influence of particular conveying parameters the following results are presented using the numerical analysis described for a belt width of 1.2m, a bulk solid with a

(1) DEM refers to Discrete Element Modeling, where the path of each individual granular particle within a system is computed to establish flow characteristics.
(2) CSIRO is Australia's Commonwealth Scientific and Industrial Research Organisation.

density of 1000kg/m^3, and a friction angle between the bulk solid and the belt of $\phi_w = 30^o$. The results cover a range of bulk solid kinematic internal friction angles, idler spacings, sag ratios and belt speeds. The results demonstrate the influence that each parameter has on the magnitude of the bulk solid flexure resistance so as to better inform the conveyor designer of the significance of particular parameters at the design stage.

While the conveyor designer typically has little control over the properties of the bulk solid being conveyed, and in particular the internal friction angle, it is still worth noting its influence on the bulk solid flexure resistance. Figure 6 shows the internal friction angle versus the bulk solid flexure resistance for a range of idler spacings. As the internal friction angle increases the ratio between the passive and active stress factors also increases. This has the effect of increasing both the longitudinal and lateral components of the bulk solid flexure resistance, which are calculated from the difference between the work done during the passive and active stress states.

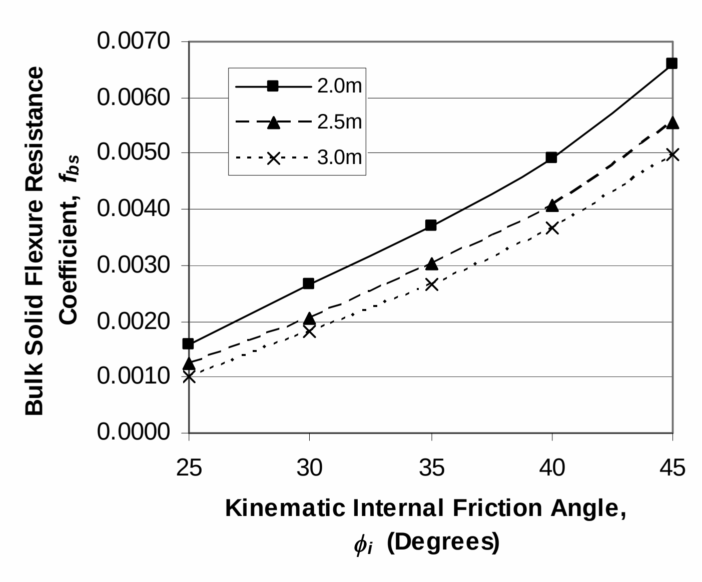

Figure 6: Calculated bulk solid flexure resistance coefiicient versus kinematic internal friction angle for a range of idler spacings. (belt speed = 5m/s, belt width = 1.2m, sag ratio = 2%, ρ = 1000kg/m^3, $\phi_w = 30^o$).

Also of interest is the reduction in the bulk solid flexure resistance coefficient with increasing idler spacing. This occurs since the magnitude of the flexure resistance per idler set only increases marginally with idler spacing since in the present example the sag ratio is maintained at 2%. Consequently, the flexure resistance force per unit length decreases with increasing idler spacing providing belt tension is increased accordingly to maintain 2% sag.

Figure 7 shows the influence of keeping the idler spacing constant and varying the sag ratio. As expected increasing sag results in higher bulk solid flexure resistance since the belt and therefore the bulk solid undergoes greater relative movement that results in higher frictional losses. Furthermore, as the internal friction of the bulk solid becomes greater there is a relative increase in bulk solid flexure resistance.

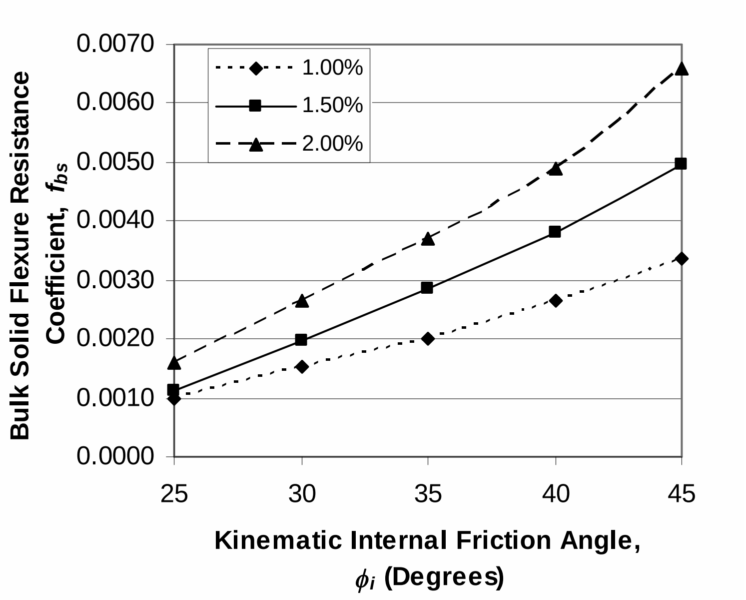

Figure 7: Calculated bulk solid flexure resistance coefficient versus kinematic internal friction angle for a range of sag ratios. (belt speed = 5m/s, belt width = 1.2m, idler spacing = 2m, ρ = 1000kg/m^3, ϕ_w = 30^o).

Figure 8 shows the influence of belt speed on the bulk solid flexure resistance. As the belt speed increases the bulk solid flexure resistance also increases since the transition between the active and passive stress states takes place at a location greater than 50% of the idler spacing. The increasing flexure resistance occurs for each of the sag ratios shown and is slightly more pronounced with higher sag ratios.

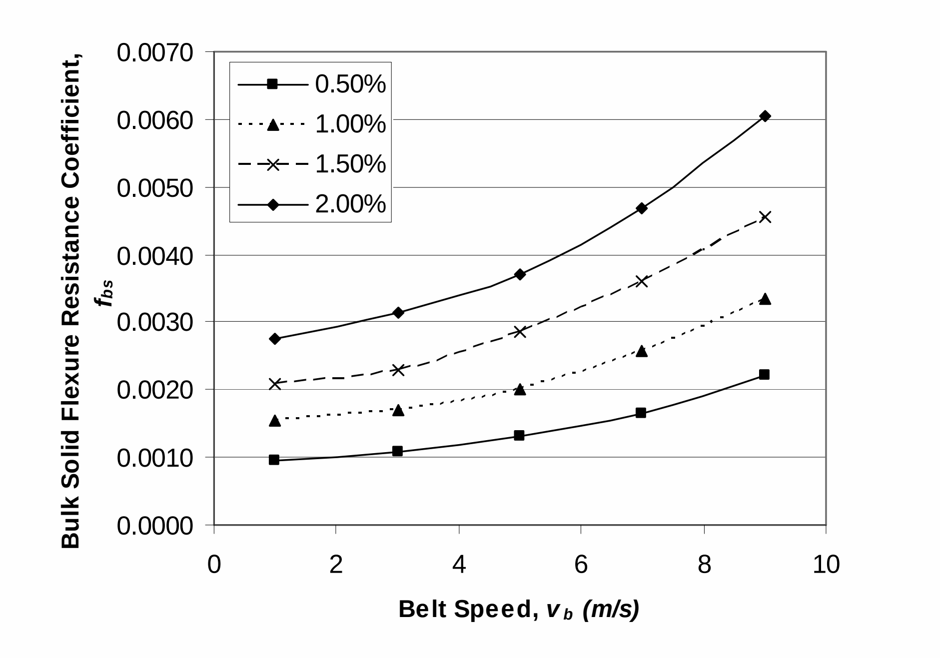

Figure 8: Calculated bulk solid flexure resistance coefficient versus belt speed for a range of sag ratios. (belt width = 1.2m, idler spacing = 2m, ϕ_i = 35^o, ρ = 1000kg/m^3, ϕ_w = 30^o).

Given the influence of particular conveyor parameters on the bulk solid flexure resistance it is clear that the conveyor designer has control over many of these variables at the design stage. The selection of these variables should be made with consideration to the other components of the main resistance in addition to the total life cycle cost of the conveyor installation. Additionally, while the present discussion has been limited to static conveyor design in practice this particular methodology may also be incorporated into a dynamic analysis program where the bulk solid flexure resistance is expressed as a function of belt tension and belt speed.

5. EXPERIMENTAL VERIFICATION

An important aspect of the development of the model was experimental verification of the results. This was undertaken using the test facilities of TUNRA Bulk Solids Research Associates(3). As indicated in Eqns (1) and (2) the active and passive stress factors are dependent on the wall friction angle, ϕ_w between the conveyor belt and the bulk solid, and the kinematic internal friction angle of the bulk solid, ϕ_i. These properties are best measured using representative samples of both the conveyor belt and the bulk solid. Figure 9 shows a picture of TUNRA's large wall friction tester which is used to experimentally measure ϕ_w. The wall friction tester has a 300mm shear cell which holds the bulk solid and shears the sample against a 400mm x 400mm sample of conveyor belt. The inverted shear cell enables larger, more representative particles to be tested, rather than what is possible in a conventional Jenike wall friction tester. Figure 10 shows TUNRA's large Jenike shear cell tester which is used to measure ϕ_i. Once again the main advantage is the ability to test samples containing larger particle sizes.

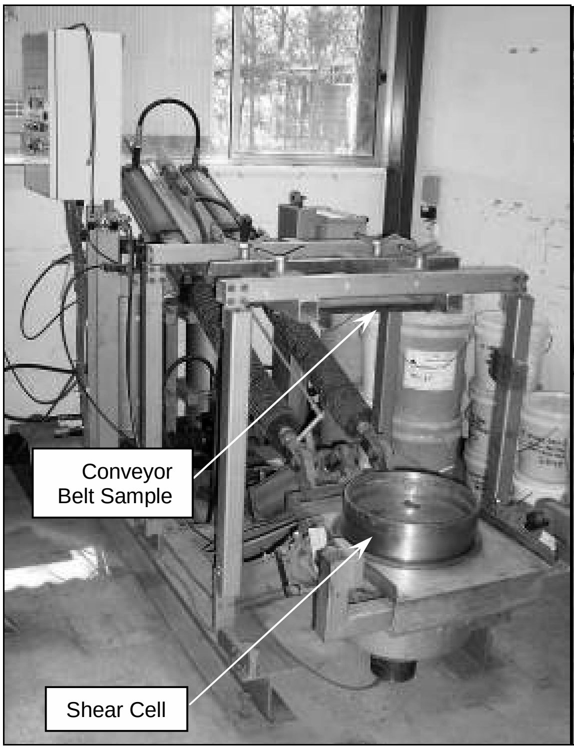

Figure 9: Large inverted wall friction tester.

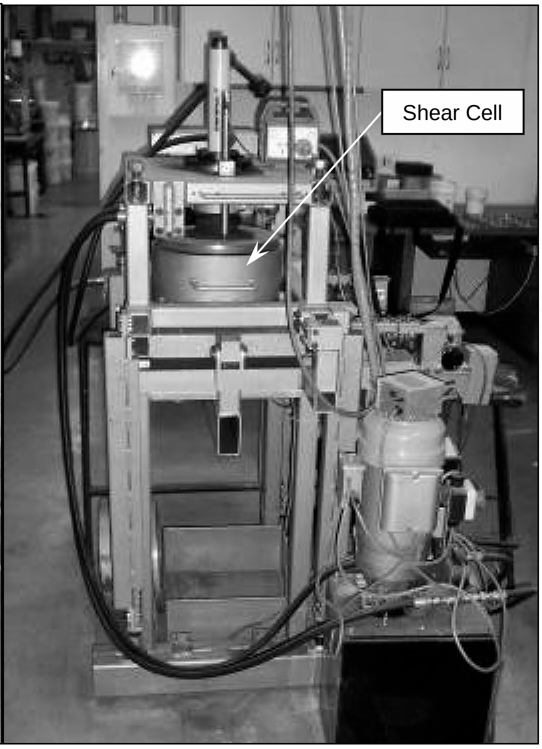

Figure 10: Large Jenike shear cell tester.

In addition to the bulk solid properties the longitudinal and transverse bending moduli of the conveyor belt are also required to be measured experimentally for use in the belt deflection model. These properties are measured experimentally using conventional materials testing facilities. Given the experimentally measured belt and bulk solid properties they can be used

(3) TUNRA Bulk Solids Research Associates is an industrial consulting division of The University of Newcastle.

directly in the model to calculate the flexure resistance due to the bulk solid. Typically the analysis is undertaken for a range of belt tensions to simulate positions along the length of the conveyor belt.

To experimentally verify the theoretical model the author developed an instrumented idler set to measure the total motion resistance at a given point in a belt conveyor. To determine the contribution of the bulk solid flexure resistance the other main resistance components were measured and then subtracted from the total resistance (see Wheeler [4]).

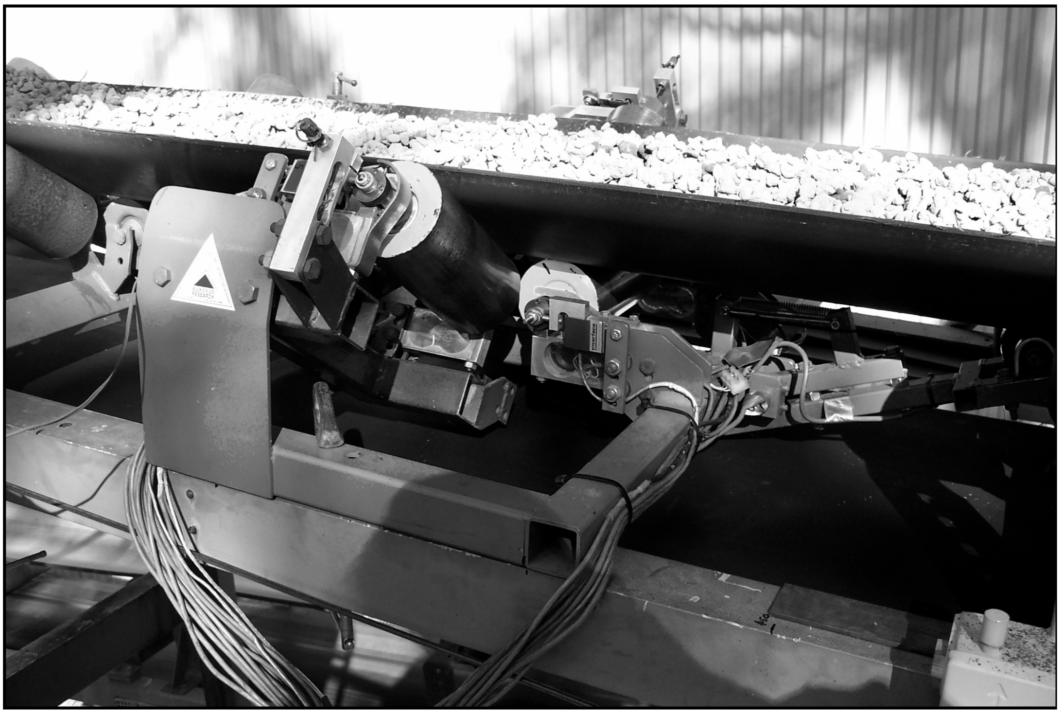

Figure 11: Main resistance measurement apparatus (Ref: Wheeler [4]).

Tests were undertaken using a fabric conveyor belt to amplify the bulk solid flexure. The dependence of the bulk solid flexure resistance on the internal angle of friction was amplified by testing bulk solids with similar bulk densities and friction angles with the conveyor belt, but significantly different internal friction angles. The results showed an increase in flexure resistance with the internal friction angle which correlated well with the theoretical analysis.

6. CONCLUSION

When a bulk solid is transported on a belt conveyor flexure resistance occurs between successive idler sets as the bulk solid undergoes transverse and longitudinal displacement due to the sag of the belt. A theoretical model was described to calculate the bulk solid flexure resistance and results presented to highlight the importance of a number of conveyor parameters. Knowing the influence of particular parameters it is clear that the conveyor designer has control over many of these at the design stage, and as a result their selection should be carefully made with consideration to the other components of the main resistance and the total life cycle cost of the conveyor installation.

7. REFERENCES

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8. CURRICULUM VITAE

Dr Wheeler is currently a Lecturer in the School of Engineering at the University of Newcastle, Australia. He worked as a Mechanical Engineer for BHP Billiton for 11 years and then as a Research Fellow at the Centre for Bulk Solids and Particulate Technologies for 4 years. He was appointed as a Lecturer in Mechanical Engineering in 2002 and undertakes industrial consulting activities through TUNRA Bulk Solids Research Associates.